SISSA-118/96/EP

Higher Derivative F-Terms in $N = 2$ strings

Jose F. Morales[b] and Marco Serone[a,b]

[a]*Istituto Nazionale di Fisica Nucleare, sez. di Trieste, Italy*

[b]*International School for Advanced Studies,ISAS-SISSA I-34100 Trieste, Italy*

Abstract

We study a special class of higher derivative F-terms of the form $F_{g,n}W^{2g}(\Pi f)^n$ where W is the $N = 2$ gravitational superfield and Π is the chiral projector applied to a non-holomorphic function f of the heterotic dilaton vector superfield. We analyze these couplings in the heterotic theory on $K3 \times T^2$, where it is found they satisfy an anomaly equation as the well studied $F_{g,0}$ terms. We recognize that, near a point of SU(2) enhancement, a given generating function of the leading singularity of the $F_{g,n}$ reproduces the free energy of a c=1 string at an arbitrary radius R. According to the $N = 2$ heterotic-type II duality in 4d, we then study these couplings near a conifold singularity, using its local description in terms of intersecting D-5-branes. In this context, it turns out that there exists, among the other states involved, a vector gauge field reproducing the heterotic leading singularity structure.

1. Introduction and Summary

The considerable progress in the non-perturbative understanding of string theories have established strong links between apparently different strings as different limits of more fundamental theories [1, 2]. Among other dualities, the $N = 2$ string in four dimensions, realized by dual pairs constructed from Calabi-Yau and $K3 \times T^2$ compactifications of the type II and heterotic string respectively [3], have received special interest as an intermediate step toward more realistic $N = 1$ models.

The appearence of singularities in the vector moduli space is a common fact of these $N = 2$ vacua. They are well understood as loci where additional charged particles become massless. While in the heterotic vacua a detailed analysis of these singularities can be done, since the states which become massless are elementary states, in the type II side the conifold singularities are related to solitonic branes, charged under R-R gauge fields, wrapped around vanishing cycles of the Calabi-Yau manifold [4] and our possibilities of study are then more limited. The Polchinski proposal of an exact conformal field theory for these solitonic objects [5, 6] as Dirichlet branes has given strength to this proposal and also offered us a device to perform string computations. Rather amazingly, D-branes have been used also to construct new vacuum configurations that locally describe the conifold in terms of intersecting five-branes [7], where the hypermultiplet becoming massless is a fundamental string state, giving us the opportunity of performing perturbative computations.

In this paper we will exploit this picture in order to get a better understanding of the nature of higher derivative terms in string theory. The study of higher derivative F-terms is crucial if one wants to establish a string-string duality beyond the effective action level. Although in general the control of these terms is very difficult, a certain class of them, those of the form $F_g(Z)W^{2g}$, with W the $N = 2$ gravitational superfield, have been proven to be controlled by holomorphic anomaly equations for both $N = 2$ Type II vacua and some heterotic dual models [8, 9]. Moreover, these F_g couplings have been also related

to topological partition functions of twisted sigma models on Calabi-Yau manifolds. The singularity structure in both side were also studied and it was found an exciting connection with the $c = 1$ string at the self-dual radius [10].

After a brief section in which we review some generalities of higher derivative F-terms, we complete in section three the study of the F_g couplings on the heterotic side, showing that for an arbitrary model $O(2, n_V)$, n_V being the number of vector multiplets (the $O(2, 1)$ case was studied in [11] arriving to similar results) they are controlled by an holomorphic anomaly equation of the same kind as the one observed for type II strings in the weak coupling limit. We should point out that the computation of this anomaly equation for $N = 2$ heterotic vacua is independent of any statement of duality and valid even in the case of heterotic models which lack of a type II dual.

Section three is devoted to the study of a sequence of generalized F-terms for the $N = 2$ heterotic string. They are of the form $F_{g,n}W^{2g}(\Pi f)^n$ where W is again the gravitational superfield and Π is the superconformal $N = 2$ chiral projector acting on a general function f of the complex vector superfields the heterotic dilaton belongs to. We prove the existence of an anomaly equation for this new sequence. The singularity structure near codimension one locus of symmetry enhancement is studied and it reproduces the free energy of a $c = 1$ string at a radius R.

Section four explores the type II picture of this new sequence of F-couplings. After providing a local D-brane description of the conifold, we analyze different gauge fields looking for a candidate of the corresponding dual vectormultiplet of the dilaton, gaining some insights about the duality map. The match between the corresponding structure is realized only in the leading singularity limit. Away from this limit the computation requires much more work. In the last section we give our conclusions.

2. Higher Derivative F-Terms

In this section we briefly review some important aspects of F-couplings and construct a generalized sequence of them relevant for the discussions below.

The so called F-terms are constructed from the F-component of a chiral superfield. The most extensively studied ones are of the type

$$I_g = F_g(X)\, W^{2g}|_{F-comp} \tag{2.1}$$

where W is the (weight 1) Weyl superfield [1],

$$W^{ij}_{\mu\nu} = F^{ij}_{\mu\nu} - R_{\mu\nu\lambda\rho}\,\theta^i\sigma_{\lambda\rho}\theta^j + \ldots, \tag{2.2}$$

that is anti-self-dual in its Lorentz indices and antisymmetric in the indices i, j labeling the two supersymmetries; $R_{\mu\nu\lambda\rho}$ is the anti-self-dual Riemann tensor, while $F^{ij}_{\mu\nu}$ is the (anti-self-dual) graviphoton field strength. $F_g(X)$ is an analytic function of the $N = 2$ chiral superfields X^I (of Weyl weight 1):

$$X^I = \hat{X}^I + \frac{1}{2}\widehat{F}^I_{\lambda\rho}\epsilon_{ij}\theta^i\sigma_{\lambda\rho}\theta^j + \ldots, \tag{2.3}$$

where $\hat{X}^I$, $\widehat{F}^I_{\lambda\rho}$ are the scalar components and the anti-self-dual vector field strengths of X^I.

The moduli dependence of these couplings (which one would naively expect to be holomorphic) has been proven to be controlled by an holomorphic anomaly equation for the $N = 2$ Calabi-Yau compactification of type II strings [8] and for the O(2,1) conjectured heterotic dual model [11]. The generalization to an arbitrary $O(2, n)$ heterotic model will be the subject of the next section. It was pointed out in [11] that there are holomorphic ambiguities not captured by these anomaly equations. A discussion of the holomorphic structure is also included in that section. In both cases, type II and heterotic vacua, the

[1] For a general discussion of $N = 2$ supergravity see ref.[12].

leading singularities have been studied and are in perfect agreement with the Strominger's interpretation of these singularities.

More general F-terms can be constructed by including superfields which are chiral projections of complex vector superfields. The superconformal chiral projection Π is a generalization of the $\bar{D}^{i^2}\bar{D}^{j^2}$ of the rigid supersymmetry [2]. The new sequence of F-terms are of the type

$$I_{g,n} = \tilde{F}_{g,n}(X)\,(\Pi f(X,\bar{X}))^n\,W^{2g}|_{F-comp}, \tag{2.4}$$

where $f(X,\bar{X})$ is an arbitrary function of the complex vector superfields while $\tilde{F}_{g,n}$ is an analytic one.

The study of these kind of interactions will occupy the main part of this paper. The relevant amplitudes are given by a bunch of $2n$ vector fields from the dilaton multiplet beside the usual $2g-2$ graviphotons and two gravitons in the heterotic string, corresponding to the following term in the effective action:

$$I_{g,n} = F_{g,n}F_d^{2n}\{g(R^2)(F_f^2)^{g-1} + 2g(g-1)(RF_f)^2(F_f^2)^{g-2}\} + ... \tag{2.5}$$

where $R^2 = R_{abcd}R^{abcd}$, $F^2 = F_{ab}F^{ab}$ and $(RF)^2 = (R_{abcd}F^{cd})(R^{abef}F_{ef})$ and the subindices d, f refer to the vector in the dilaton supermultiplet and the graviphoton respectively. It is understood that R_{abcd} and F_f represent the anti-self-dual parts of the Riemann tensor and graviphoton, while F_d is the self-dual part of the field strength corresponding to the dilaton.

In the D-brane description it will be convenient to consider an amplitude involving the term with $2g$ graviphotons, $2n$ field strenghts and two gauge fields sitting in the vector multiplet of the modulus μ. The relevant piece of the effective action is then

$$I_{g,n}|_F = \partial_\mu^2 F_{g,n}F_\mu^2 F_d^{2n}F_f^{2g} + ... \tag{2.6}$$

[2]For $N=1$ case, Π was worked out in detail in reference [13] which can be consulted for details

3. Holomorphic Anomaly Equations for $O(2,n)$ Case

We study now the moduli dependence of F_g-terms for arbitrary $N=2$ heterotic vacua. More precisely, we consider $N=2$ heterotic models with rank $n+2$ where, apart from the dilaton, the scalars in vector multiplets sit in the coset $O(2,n)/O(2)\times O(n)$ modulo discrete identifications that define the duality group. The case $n=1$ was discussed in detail in [11], and the generalization for $n=2$ is straightforward. Here we will show that for any n (for heterotic models n is bounded by 22), the couplings F_gW^{2g} exist and satisfy a recursion relation, following from the holomorphic anomaly equation, that coincides with the expected recursion relation from the dual type II model in the weak coupling limit.

As was pointed out in the last section, the F_g couplings arise from heterotic amplitudes A_g's involving two gravitons and $(2g-2)$ graviphotons. In [11] these amplitudes were computed at one loop level in heterotic string and the result is in the following expression for F_g:

$$F_g = -\frac{(4\pi i)^{g-1}}{4\pi^2}\int \frac{d^2\tau}{\tau_2^3}\frac{\tau_2^{2g}}{\bar{\eta}^3}G_g(\tau,\bar{\tau})\sum_C C_c(\bar{\tau})\sum_{(P_L,P_R)\in\mathbf{\Gamma_c}}(e^{K_0/2}P_L)^{2g-2}q^{\frac{1}{2}|P_L|^2}\bar{q}^{\frac{1}{2}P_R^2}, \qquad (3.1)$$

where τ is the Teichmuller parameter of the world-sheet torus, q is $e^{2\pi i\tau}$, P_L, $\bar{P}_L$ and $\vec{P}_R$ are the left and right moving momenta sitting in an $n+2$ real dimensional lattice Γ_c with the $O(2,n)$ inner product $\frac{1}{2}(P_L\bar{P}'_L+\bar{P}_LP'_L)-\vec{P}_R\vec{P}'_R$. In general the lattice of momenta corresponding to vector multiplets with the inner product defined above is not self-dual. As a result, world sheet modular invariance implies that the vectors in the dual lattice must appear in the spectrum. This dual lattice splits into several conjugacy classes labelled by Γ_c. Each of these classes are coupled to different blocks of the remaining conformal field theory ($c=6, \bar{c}=22-n$) and $C_c(\bar{\tau})=(-1)^Fq^{L_0-c/24}\bar{q}^{\bar{L}_0-\bar{c}/24}$ in this block. $C_0(\bar{\tau})$ has the expansion $C_0(\bar{\tau})=\bar{q}^{-1}(1-n_H\bar{q})$ where n_H is the number of neutral massless hypermultiplets at generic point in the $O(2,n)/(O(2)\times O(n))$ moduli space. $G_g(\tau,\bar{\tau})$ is the normalized correlator $G_g(\tau,\bar{\tau})=\frac{1}{(g!)^2}(\frac{1}{\tau_2})^{2g}\langle\prod_{i=1}^g\int d^2x_iZ^1\bar{\partial}Z^2(x_i)\prod_{j=1}^g\int d^2y_j\bar{Z}^2\bar{\partial}\bar{Z}^1(y_j)\rangle$ where Z^1, Z^2

are the two complex bosons representing the four space-time coordinates. In [11] the generating function for G_g was shown to be

$$G(\lambda,\tau,\bar{\tau}) \equiv \sum_{g=1}^{\infty} \lambda^{2g} G_g(\tau,\bar{\tau}) = \left(\frac{2\pi i\lambda\bar{\eta}^3}{\bar{\Theta}_1(\lambda,\bar{\tau})}\right)^2 e^{-\frac{\pi\lambda^2}{\tau_2}}, \tag{3.2}$$

which implies the following recursion relation for G_g:

$$\partial_\tau G_g = -\frac{i\pi}{2}\frac{1}{\tau_2^2} G_{g-1} \tag{3.3}$$

In the following we will also need the precise form of P_L for the $O(2,n)$ case. Let us parametrize the coset $O(2,n)/O(2)\times O(n)$ by the n variables T, U, y_α, $\alpha = 3, ..., n$, where T, U are respectively the (complexified) Kähler class and complex structure of the two-dimensional target space torus, and y_α represent the $n-2$ Wilson lines. The classical prepotential for this model is given by

$$F = S(TU - y^2), \tag{3.4}$$

and the corresponding classical Kähler potential is

$$K_{tree} = -\log\ i(S-\bar{S})[(T-\bar{T})(U-\bar{U}) - (y_\alpha - \bar{y_\alpha})^2]. \tag{3.5}$$

where, as usual, S is the complex scalar associated to the dilaton, with the normalization $< S >= \frac{\theta}{\pi} + i\frac{8\pi}{g_s^2}$. P_L is just the $N=2$ central charge given as

$$P_L = e^{K_0/2}(n_1 + n_2(\bar{T}\bar{U} - \bar{y}^2) + m_1\bar{T} + m_2\bar{U} + k_\alpha\bar{y_\alpha}), \tag{3.6}$$

and

$$K_0 \equiv -\log[(T-\bar{T})(U-\bar{U}) - (y_\alpha - \bar{y_\alpha})^2]. \tag{3.7}$$

We are ready now to find the holomorphic anomaly equation satisfied by the F_g's. Taking antiholomorphic derivatives of (3.1), we find that in order to get the generalized recursion relations for arbitrary rank n, the holomorphic and antiholomorphic derivatives of $P_L, \bar{P}_L$ must be related in a highly non-trivial way through

$$\partial_{\bar{i}}(e^{K_0/2}P_L) = A_{\bar{i}j}\partial_j(e^{K_0/2}\bar{P}_L) \tag{3.8}$$

After some tedious algebra one can show that this algebraic system of $n(n+2)$ equations for the n^2 variables has solution, which is related to the prepotential (3.4) through

$$A_{\bar{i}j} = C_{\bar{i}\bar{l}\bar{s}} G^{\bar{l}j} \tag{3.9}$$

where

$$C_{\bar{i}\bar{j}\bar{k}} = \partial_{\bar{i}}\partial_{\bar{j}}\partial_{\bar{k}}\bar{F} \tag{3.10}$$

are the Yukawa couplings, and $G^{\bar{l}j}$ the inverse of the metric.

Substituting this expression in the antiholomorphic derivatives of F_g, one can easily show that for $g > 1$

$$\begin{aligned} \partial_{\bar{i}} F_g &= \frac{i}{4\pi^3}(4\pi i)^{g-1} e^K A_{\bar{i}j} \int d^2\tau G_g(\tau,\bar{\tau}) \times \\ \partial_\tau & [\frac{\tau_2^{2g-3}}{\bar{\eta}^3} \sum_C C_c(\bar{\tau}) \sum_{(P_L,P_R)\in\Gamma_c} (e^{K_0/2}P_L)^{2g-4} \partial_j (q^{\frac{1}{2}|P_L|^2} \bar{q}^{\frac{1}{2}P_R^2})], \end{aligned} \tag{3.11}$$

where

$$i, j = T, U, y_\alpha$$

Now we can perform a partial integration with respect to τ. The boundary term vanishes for a generic point on the moduli space away from the singularities. The only nonvanishing contribution appears when ∂_τ acts on G_g. Using now eq.(3.3) and the expression (3.9) for $A_{\bar{i}j}$ one obtains

$$\partial_{\bar{i}} F_g = 2\pi i e^{2K_0} C_{\bar{i}\bar{l}\bar{s}} G^{\bar{l}j} D_j F_{g-1}, \tag{3.12}$$

where D_j is the Kähler covariant derivative, i.e.

$$D_j F_g = (\partial_j - 2(g-1)\partial_j K) F_g, \tag{3.13}$$

since F_g has Kähler weight $2g-2$. Note that similar relations for the holomorphic anomaly equations have been found in [14] purely from $N=2$ supergravity arguments.

The expression (3.12) coincides with the expected recursion relation satisfied by F_g for Type II in the Kähler moduli limit corresponding to $S \to \infty$. We should point out

that these holomorphic anomalies are, like in the type II side, related to non-localities or massless states that contribute in the boundaries of the moduli space of Riemman surfaces that explains the origin of the recursion relations from handle and dividing degenerations. As was pointed out in the last section there is an holomorphic ambiguity which is not captured by these anomaly equations. The determination of the holomorphic structure of F_g for the Type II string is extremely difficult since it involves integration of the topological partition function of the twisted Calabi-Yau sigma models over the moduli space of genus-g Riemann surfaces. However, one can compare the leading singularities in F_g near the loci. This analysis was done in [11] for the rank 3 case. The generalization of these results is straightforward. The local coordinate that goes to zero at the locus is taken to be $\mu = \sqrt{i/\pi}\exp(-K_0/2)Z$, where Z is the central charge of the $N = 2$ supersymmetry algebra. Again two states become massless at each locus $T = U$, $TU = y^2$ and $y_\alpha = 0$, corresponding to the charges $m_1 = -m_2 = \pm 1$,$n_2 = \pm 1$ and $k_\alpha = \pm 1$ respectively (all the non-specified charges are zero). The leading coefficient reproduces (-2) times the free enegy of the $c = 1$ string [10] as we expected from the discussion above. This provides further support to the Strominger's interpretation of these singularities and conclude our study of the F_g terms.

4. $F_{g,n}$ terms in $N = 2$ heterotic vacua

We have seen previously how it is possible to construct more general F-terms that include chiral projections of complex vector superfields. Let us then begin the study of the $F_{g,n}$ terms defined by (2.5) for the $N = 2$ heterotic string. The computation generalizes the result of [11], and it is very similar to that of the F_g's, therefore we will stress only the differences. The relevant amplitude is given by

$$A_{g,n} = \langle V_h(p_1^+)V_h(p_1^-)\prod_{i=1}^{g-1} V_f(p_1^{+(i)})\prod_{j=1}^{g-1} V_f(p_1^{-(j)})\prod_{l=1}^{n} V_d(p_1^{+(l)})\prod_{m=1}^{n} V_d(p_1^{-(m)})\rangle$$

$$= \quad (p_1^+)^2(p_1^-)^2\prod_{i,j=1}^{g-1}p_1^{+(i)}p_1^{-(j)}\prod_{l,m=1}^{n}p_1^{+(l)}p_1^{-(m)}(g!)^2(n!)^2F_{g,n}. \tag{4.1}$$

where we have used the complex notation:

$$\begin{aligned} a_1^{\pm} &= \frac{1}{\sqrt{2}}(a^1 \pm ia^2) \\ a_2^{\pm} &= \frac{1}{\sqrt{2}}(a^3 \pm ia^4) \end{aligned} \tag{4.2}$$

for every spacetime vector a^μ.

We have chosen the kinematical configuration where $p_1^+ \neq 0$, $p_2^+ = p_1^- = p_2^- = 0$ for half of the operators and $p_1^- \neq 0$, $p_1^+ = p_2^+ = p_2^- = 0$ for the other half. The vertex operators, in this configuration for the anti self-dual part of the graviton and graviphoton, and the self-dual part of the vector superpartner of the dilaton are respectively:

$$\begin{aligned} V_h(p_1^{\mp}) &= (\partial Z_2^{\pm} - ip_1^{\mp}\chi_1^{\pm}\chi_2^{\pm})\bar\partial Z_2^{\pm}e^{ip_1^{\mp}Z_1^{\pm}} \\ V_f(p_1^{\mp}) &= (\partial X - ip_1^{\mp}\chi_1^{\pm}\Psi)\bar\partial Z_2^{\pm}e^{ip_1^{\mp}Z_1^{\pm}} \\ V_d(p_1^{\mp}) &= (\partial X - ip_1^{\mp}\chi_1^{\pm}\Psi)\bar\partial Z_2^{\mp}e^{ip_1^{\mp}Z_1^{\pm}} \end{aligned} \tag{4.3}$$

where X is the complex coordinate of the left-moving torus T^2 and Ψ is its fermionic partner with $U(1)$ charge $+1$.

As in [11], the full amplitude can be evaluated in the odd spin structure, where one of the operators should be inserted in the (-1)-ghost picture due to the presence of a Killing spinor on the world sheet torus and a picture changing operator must be inserted to take care of the world-sheet gravitino zero mode. Taking one of the field strength in the (-1) ghost picture which comes with a Ψ, and the $e^\phi\bar\Psi\partial X$ part of the picture changing operator, we soak up the zero mode for $\Psi, \bar\Psi$. From the remaining $(2g+2n-3)$ gauge fields in the (0)-ghost picture only the terms involving ∂X survive since the Ψ's cannot be contracted with anything. Together with the ∂X appearing in the picture changing operator, they

provide a total of $(2g+2n-2)$ ∂X's which contribute only through their zero modes. We then take the zero momentum limit after extracting one power of momentum from each vertex operator. Including the left moving part, we arrive to a similar expression as (3.1) substituting g by $g+n$ and $G_g(\tau,\bar\tau)$ by

$$\begin{aligned} G_{g,n}(\tau,\bar\tau) &= \frac{1}{(g!)^2(n!)^2}(\frac{1}{\tau_2})^{2g+2n}\langle \prod_{i,j=1}^{g}\int d^2x_i Z_1^+\bar\partial Z_2^+(x_i)\int d^2y_j Z_1^-\bar\partial Z_2^-(y_j) \\ &\quad \prod_{l,m=1}^{n}\int d^2x_l Z_1^-\bar\partial Z_2^+(x_l)\int d^2y_m Z_1^+\bar\partial Z_2^-(y_m)\rangle \end{aligned} \tag{4.4}$$

In order to evaluate these correlation functions it is convenient to introduce the following generating function:

$$\begin{aligned} G(\lambda,\lambda',\tau,\bar\tau) &\equiv \sum_{g=1}^{\infty}\lambda^{2g}\lambda'^{2n}G_{g,n}(\tau,\bar\tau) = \\ &= \frac{\int\prod_{i=1,2}DZ_i^{\pm}exp(-S_0-S_f-S_d)}{\int\prod_{i=1,2}DZ_i^{\pm}exp(-S_0)} \end{aligned} \tag{4.5}$$

where S_0 is the free action

$S_0 = \sum_{i=1,2}\frac{1}{\pi}\int d^2x(\partial Z_i^+\bar\partial Z_i^- + \partial Z_i^-\bar\partial Z_i^+)$ and S_f, S_d being the generators of correlators involving graviphotons and vectorpartners of the dilaton: $S_f = \int d^2x\frac{\lambda}{\tau_2}(Z_1^+\bar\partial Z_2^+ + Z_1^-\bar\partial Z_2^-)$, $S_d = \int d^2x\frac{\lambda'}{\tau_2}(Z_1^-\bar\partial Z_2^+ + Z_1^+\bar\partial Z_2^-)$ respectively.

The computation of the correlators reduce then to the evaluation of this gaussian. Using the ζ-regularization, the results of the Appendix of [11] inmediately generalize to

$$G(\lambda,\lambda',\tau,\bar\tau) = \left(\frac{2\pi i\lambda^+\bar\eta^3}{\bar\Theta_1(\lambda^+,\bar\tau)}\right)\left(\frac{2\pi i\lambda^-\bar\eta^3}{\bar\Theta_1(\lambda^-,\bar\tau)}\right)e^{-\frac{\pi(\lambda^2+\lambda'^2)}{\tau_2}} \tag{4.6}$$

where $\Theta_1(z,\tau)$ is the odd theta-function and $\lambda^\pm = \lambda\pm\lambda'$.

Two interesting points can be noticed in this generalization. The first is that again the generating function satisfies a differential equation

$$\partial_\tau G(\lambda,\lambda'\tau,\bar\tau) = -\frac{i\pi}{2}\frac{\lambda^2+\lambda'^2}{\tau_2^2}G(\lambda,\lambda'\tau,\bar\tau) \tag{4.7}$$

which define a recursion relation for the coefficients $G_{g,n}$ of the eq.(4.5)

$$\partial_\tau G_{g,n} = -\frac{i\pi}{2}\frac{1}{\tau_2^2}(G_{g-1,n} + G_{g,n-1}) \tag{4.8}$$

Substituting (4.8) in (3.11) and integrating by parts as before we found that the new sequence satisfies also an holomorphic anomaly equation

$$\partial_{\bar{i}} F_{g,n} = 2\pi i e^{2K_0} C_{\bar{i}\bar{l}\bar{s}} G^{\bar{l}j} D_j (F_{g-1,n} + F_{g,n-1}), \tag{4.9}$$

We see that these higher derivative terms behave much more like the F_g's, motivating our search for similar objects in the type II side, the subject of the next section.

The second observation refers to the singularity structure of these F-couplings near a codimension one locus of enhanced symmetry described by a local coordinate $\mu = \sqrt{\frac{i}{\pi}} exp(-\frac{K_0}{2}) P_L$ which goes to zero. After some simple algebra and rescaling of variables, the leading term for $F_{g,n}$ is given by the coefficient of $\lambda^{2g}\lambda'^{2n}$ in the expansion of

$$(\mu)^{2-2g-2n} \int d\tau_2 \tau_2^{2g+2n-3} (\frac{\pi\lambda^+}{sin\pi\lambda^+})(\frac{\pi\lambda^-}{sin\pi\lambda^-}) e^{-\tau_2} \tag{4.10}$$

We recognize in this expression the free energy of the $c = 1$ string with cosmological constant μ on a radius $R = \lambda^-/\lambda^+$. This provide further evidence of the profound connection of the $c = 1$ string as topological description of conifold singularities and deserves a more detailed study.

5. D-brane realization

In the local description of the conifold singularity in terms of intersecting D-five-branes constructed by [7], the F_g couplings have been recently studied in [15], where it was found the expected singularity structure. This result strongly supports the description given by [7] for the conifold, so we look, in this picture, for a gauge field that reproduces the behaviour of the new sequence of $F_{g,n}$ couplings, computed previously in the heterotic

side. A careful comparison of the corresponding singular structures allows us to extract some useful information about the involved duality map. As first step, let us review the construction of [7, 15]. Basically, conifold singularities are mimiced by two non-parallel D-branes lying, say, in

$$
\begin{aligned}
&(x_4, x_5, x_8, x_9) = 0 \\
&(x_6, x_7, x_8, x_9) = v
\end{aligned}
\tag{5.1}
$$

For $v = 0$ they intersect on a $3+1$ dimensional space identified with the spacetime. The chosen configuration breaks $\frac{3}{4}$ of the supersymmetry leading to $N = 2$ SUSY in four dimensions. The conserved supercharges are given by [16] the intersection of

$$Q_A + (\gamma^6\gamma^7\gamma^8\gamma^9)_A^B \tilde{Q}_B \tag{5.2}$$

and

$$Q_A + (\gamma^4\gamma^5\gamma^8\gamma^9)_A^B \tilde{Q}_B \tag{5.3}$$

where $A, B = 1, ...16$ and $Q, \tilde{Q}$ are the left and right charges respectively. More explicitly, they are

$$Q_1^\alpha + \tilde{Q}_1^\alpha; \quad \text{and} \quad Q_2^\alpha - \tilde{Q}_2^\alpha.$$

together with the corresponding dotted generators, of course, where

$$Q_k^\alpha \equiv \int e^{-\frac{1}{2}\phi} S^\alpha \Sigma_k \qquad k = 1, 2 \tag{5.4}$$

and ϕ is the bosonization of superghost. S^α and Σ_i are the spin fields of the four-dimensional space-time and the internal parts respectively, and they can be given explicitly in the bosonized form as follows:

$$S^\pm = e^{\pm\frac{i}{2}(\phi_1+\phi_2)}.$$

$$\Sigma_k = e^{i(\frac{3}{2}-k)(\phi_3+\phi_4)+i\frac{1}{2}\phi_5}$$

Beside the universal and closed sector we include different open sectors related to the possible boundary conditions on endpoints of the string living in the D-brane as N-N, N-D,

D-N, D-D according to which boundary condition, Neumann or Dirichlet, is chosen. The conifold singularity is then resolved by a string stretched between the two D-branes, that give rise to a massless hypermultiplet, charged under the difference of the $U(1)$ gauge fields living in the boundary.[3] The analysis of the holomorphic structure of higher derivative couplings can be done through an annulus computation of a bunch of graviphotons and gauge fields as defined by (2.6).

We shall see that there is a natural candidate vector reproducing the given behaviour near the conifold. The computation involve a bunch of $2n$ of these vectors besides 2g graviphotons and two boundary gauge fields. The vertex operators in the (-1)-ghost picture and in the kinematical configuration (in the notation (4.2)) where half of the operators have $p_1^+ \neq 0, p_1^- = p_2^\pm = 0$ and the rest $p_1^- \neq 0, p_1^+ = p_2^\pm = 0$, are given by:

$$\begin{aligned}
V_f(p_1^\mp) &= [(e^{-\phi}\psi_5^+)(\bar{\partial}Z_2^\pm + ip_1^\mp\tilde{\psi}_1^\pm\tilde{\psi}_2^\pm) - (\partial Z_2^\pm + ip_1^\mp\psi_1^\pm\psi_2^\pm)(e^{-\tilde{\phi}}\tilde{\psi}_5^\pm) \\
&+ ip_1^\mp e^{-1/2\phi}S^\pm\Sigma_1 e^{-1/2\tilde{\phi}}\tilde{S}^\pm\tilde{\Sigma}_2 - ip_1^\mp e^{-1/2\phi}S^\pm\Sigma_2 e^{-1/2\tilde{\phi}}\tilde{S}^\pm\tilde{\Sigma}_1]e^{ip_1^\mp Z_1^\pm}. \\
V_d(p_1^\mp) &= [(e^{-\phi}\psi_5^+)(\bar{\partial}Z_2^\mp + ip_1^\mp\tilde{\psi}_1^\pm\tilde{\psi}_2^\mp) - (\partial Z_2^\mp + ip_1^\mp\psi_1^\pm\psi_2^\mp)(e^{-\tilde{\phi}}\tilde{\psi}_5^\pm)]e^{ip_1^\mp Z_1^\pm} \qquad (5.5) \\
V_b(p) &= \int(\partial_\tau X^\mu + ip\psi\gamma_\tau\psi^\mu)e^{ip.X}
\end{aligned}$$

The computation follows the same steps as for the F_g case considered in [15], reference that can be consulted for a more detailed analysis. In order to cancel the total superghost charge we should include $2g + 2n$ picture changing operators whose only non-vanishing contribution come from $e^{-\phi}\psi_5^-\partial Z_5^+$. Note that there is no Z_5^- in the correlation and then the Z_5^+ contributes only through its zero mode

$$Z_5^+ = \bar{\mu}\sigma + ...$$

providing an overall $\bar{\mu}^{2g+2n}$. This is the expected behaviour for the leading term of $\partial_\mu^2 F_{g,n}$ which after integration lead to the universal $\mu^{2-2g-2n}$ as in (4.10). The superghosts contri-

[3]It was argued in [15] that only this combination remains massless, while the sum adquires mass via the Cremmer-Scherk mechanism

bution cancels exactly the fermion one in the D-D direction; so we are left with a sum over the spin structures involving the remaining four directions, which defines a triality map on the $SO(8)$ weight lattice through the Riemmann identity:

$$\Sigma_\alpha(-)^{\alpha_1\alpha_2}\theta_\alpha(x_1)\theta_\alpha(x_3)\theta_\alpha(x_3)\theta_\alpha(x_4)=\theta_1(x_1')\theta_1(x_2')\theta_1(x_3')\theta_1(x_4') \tag{5.6}$$

where (α_1,α_2) are the characters of the theta functions and

$$\begin{aligned}
x_1' &= \frac{1}{2}(x_1+x_2+x_3+x_4)\\
x_2' &= \frac{1}{2}(x_1+x_2-x_3-x_4)\\
x_3' &= \frac{1}{2}(x_1-x_2+x_3-x_4)\\
x_4' &= \frac{1}{2}(-x_1+x_2+x_3-x_4)
\end{aligned}$$

are the transformed arguments.

Under this transformation the operators (5.6) get mapped to

$$\begin{aligned}
V_f(p_1^\mp) &\rightarrow (\partial_\tau Z_2^\pm+ip_1^\mp(\psi_1^\pm-\tilde\psi_1^\pm)(\psi_2^\pm-\tilde\psi_2^\pm))e^{ip_1^\mp Z_1^\pm}\\
V_d(p_1^\mp) &\rightarrow (\partial_\tau Z_2^\mp+ip_1^\mp(\psi_3^\pm\psi_4^\mp-\tilde\psi_3^\pm\tilde\psi_4^\mp))e^{ip_1^\mp Z_1^\pm}
\end{aligned} \tag{5.7}$$

while the boundary operator goes to itself. The correlations of the bosonic part give total derivatives which upon partial integration brings down one power of momentum for each vertex, matching the momentum structure of (4.1). We are left with correlators generated by a same kind of generating function (4.5) where now

$$S_f=\int d^2x\frac{\lambda}{t}(Z_1^+\bar\partial_\tau Z_2^+ + Z_1^-\bar\partial_\tau Z_2^-)$$

and

$$S_d=\int d^2x\frac{\lambda'}{t}(Z_1^-\bar\partial_\tau Z_2^+ + Z_1^+\bar\partial_\tau Z_2^-)$$

where t is the modulus of the world-sheet annulus. We observe that this perturbed action has four zero modes given by $\tilde\psi_1^\pm=\psi_1^\pm=$ constant and $\tilde\psi_2^\pm=\psi_2^\pm=$ constant, which should

be soaked by the bilinear fermion part of the inserted boundary gauge fields. Without this insertion the correlation would vanish as we expected from the absence of the corresponding term in the effective action (2.5). The mode expansion for the involved fields are:

$$\begin{aligned}
\psi^{\pm}_{1,2} &= \sum_{n\epsilon Z} \eta^{\pm(m,n)}_{1,2} e^{in\pi\sigma} e^{im\frac{2\pi}{t}\tau} \\
\psi^{\pm}_{3,4} &= \sum_{n\epsilon Z+\frac{1}{2}} \eta^{\pm(m,n)}_{3,4} e^{in\pi\sigma} e^{im\frac{2\pi}{t}\tau} \\
Z^{\pm}_{1,2} &= \sum_{n\epsilon Z} \alpha^{\pm(m,n)}_{1,2} cos(n\pi\sigma) e^{im\frac{2\pi}{t}\tau} \\
Z^{\pm}_{3,4} &= \sum_{n\epsilon Z+\frac{1}{2}} \alpha^{\pm(m,n)}_{3,4} cos(n\pi\sigma) e^{im\frac{2\pi}{t}\tau}
\end{aligned} \tag{5.8}$$

where it is understood that the sum over m always runs over the integers. For $\tilde{\psi}^{\pm}_k$ take simply $\sigma \to -\sigma$.

In contrast with the heterotic computation, we have here in general a t-dependence of the amplitude, meaning that non-BPS states are contributing as well as BPS ones, making the evaluation of the determinants not trivial at all. However, the infrared leading singularity, given by the limit $t \to \infty$, only picks up the $n = 0$ contribution. In this limit the twisted fermions decouple (due to the lack of this zero mode) and the bosonic part gives

$$\prod_{m=1}^{\infty} \left[1 - \left(\frac{(\lambda+\lambda^{'})\bar{\mu}t}{m\pi}\right)^2\right]^{-1} \left[1 - \left(\frac{(\lambda-\lambda^{'})\bar{\mu}t}{m\pi}\right)^2\right]^{-1} = \left[\frac{(\lambda+\lambda^{'})\bar{\mu}t}{\sin((\lambda+\lambda^{'})\bar{\mu}t)}\right] \left[\frac{(\lambda-\lambda^{'})\bar{\mu}t}{\sin((\lambda-\lambda^{'})\bar{\mu}t)}\right] \tag{5.9}$$

The leading singular part for $\partial^2_{\mu} F_{g,n}$ will be then given by the $\lambda^{2g}\lambda^{'2n}$ term in the expansion of

$$\int_0^{\infty} \frac{dt}{t} \left[\frac{(\lambda+\lambda^{'})\bar{\mu}t}{\sin((\lambda+\lambda^{'})\bar{\mu}t)}\right] \left[\frac{(\lambda-\lambda^{'})\bar{\mu}t}{\sin((\lambda-\lambda^{'})\bar{\mu}t)}\right] e^{-|\mu|^2 t} \tag{5.10}$$

where the term $e^{-|\mu|^2 t}$ is due to the bosonic D-D zero mode and it reproduces the same singularity structure of the heterotic locus (4.10). Is is worth while to mention that this gauge field, possible candidate dual of the corresponding heterotic vector field, is in the same multiplet of a D-D scalar.

6. Conclusions

We have studied in this paper a class of higher derivative F-terms of the form $F_{g,n}W^{2g}(\Pi f)^n$ (with W the gravitational superfield and Π the chiral projector acting on a general function f of the complex vector superfield the heterotic dilaton belongs to) coming from the $N=2$ low-energy lagrangian. It has been found that in the heterotic side, like the case for the F_g's, only BPS states contribute to the amplitude, while this is not what we found in the D-brane picture. It is important to stress here that even if the gauge field we used in the D-brane computation is not the only one reproducing the heterotic result in the leading singularity limit, it does not exist a physical state giving the heterotic structure for the $F_{g,n}$ terms away from this limit. It should be pointed out, however, that this result is not in contradiction with the conjectured heterotic-type II duality, because we compared two amplitudes computed perturbatively at 1-loop, but according to different arrangements of the perturbative expansion. There is no reason known, at least to us, for which we should expect an exact matching of the $F_{g,n}$ couplings. On the other hand, the leading behaviour is the same, according to the given duality.

We think that a further study in this direction can lead to further checks and help to clarify the nature of the $N=2$ heterotic-type II duality beyond the effective action; in particular it should be interesting to compute similar couplings taking into account vector gauge fields of general moduli.

An other point we believe should be attractive and studied better is the appearence of the free energy of the c=1 string at radius R, considering also the role played by the self-dual radius in describing the coniflod singularity. It is interesting to note that, as in [11], eqs.(4.10) and (5.10) coincide with the effective action of QED [17] in a constant background F given by $\lambda = \sqrt{(F^2 + F\tilde{F})/2}$ and $\lambda' = \sqrt{(F^2 - F\tilde{F})/2}$!

Acknowledgements

We would like to thank E. Gava and K.S. Narain for the enlightening discussions from which the idea of this paper originated and for helpful suggestions.

References

[1] C. Hull and P. Townsend, *Unity of superstring dualities*, Nucl. Phys. **B438** (1995) 109; hep-th/9410167.

[2] E. Witten, *String theory dynamics in various dimensions*, Nucl. Phys. **B443** (1995) 85; hep-th/9503124.

[3] S.Kachru and C.Vafa, *Exact Results for N=2 Compactifications of Heterotic Strings*, Nucl. Phys. **B450** (1995) 69; hep-th/9505105.

[4] A. Strominger, *Massless Black Holes and Conifolds in String Theory*, Nucl. Phys. **B451** (1995) 96; hep-th/9504090.

[5] J. Polchinski, *Dirichlet-Branes and Ramond-Ramond Charges*, Phys. Rev. Lett. **75** (1995) 4724; hep-th/9510017.

[6] J. Polchinski, *Combinatorics of boundaries in string theory*, Phys. Rev. **D50** (1994) 6041.

[7] M. Bershadsky, V. Sadov and C. Vafa, *D-Strings on D-Manifolds* , Nucl. Phys. **B463** (1996) 398; hep-th/9510225.

[8] M. Bershadsky, S. Cecotti, H. Ooguri and C.Vafa, *Kodaira-Spencer Theory of Gravity and Exact Results for Quantum String Amplitudes*, Comm. Math. Phys. **165** (1994) 311.

[9] I. Antoniadis, E. Gava, K.S. Narain and T.R. Taylor, *Topological Amplitudes in String Theory*, Nucl. Phys **B413** (1994) 162-184.

[10] D. Ghoshal and C. Vafa, *C=1 Strings as the Topological Theory of the Conifold*, Nucl. Phys **B453** (1995) 121; hep-th/9506122.

[11] I. Antoniadis, E. Gava, K.S. Narain and T. R. Taylor, *N=2 Type II-Heterotic Duality and Higher Derivative F-terms*, Nucl. Phys **B455** (1995) 109; hep-th/9507115.

[12] B. de Wit, P.O. Lauwers and A. Van Proyen, *Lagrangians of N=2 supergravity-matter systems*, Nucl. Phys. **B255** (1985) 569; E. Cremmer, C. Kounnas, A. Van Proyen, J.-P. Derendinger, S. Ferrara, B. de Wit and L. Girardello, *Vector multiplets coupled to N=2 supergravity: super-Higgs effect, flat potentails and geometric structure*, Nucl. Phys. **B250** (1985) 385.

[13] I. Antoniadis, E. Gava, K.S. Narain and T.R. Taylor, *Topological Amplitudes in Heterotic Superstring Theory*, hep-th/9604077.

[14] B. de Wit, *N=2 Electric-magnetic duality in a chiral background*, Proc. of the 29^{th} International Symposium Arhenshoop on the Theory of Elementary Particles, Buckow, 1995, published in Nucl. Pyhs. **B** (Proc. Suppl.) **49** (1996) 191, hep-th/9602060; *N=2 symplectic reparametrizations in a chiral background*, Proc. of the workshop on STU-Dualities and Nonperturbative Phenomena in Superstrings and Supergravity, Cern Theory Division, 1995; to appear in Fortschritte der Physik, hep-th/9603191.

[15] E. Gava, T.Jayaraman, K.S. Narain and M.H.Sarmadi, *D-Branes and the Conifold Singularity*, hep-th/9607131.

[16] J. Polchinski, S. Chaudhuri and C. V. Johnson, *Notes on D-Branes*, hep-th/9602052.

[17] J. Schwinger, *On gauge invariance and vacuum polarization*, Phys. Rev **82** (1951) 664.